HEAT TRANSFER IN THE FLOW OF A LIQUID-METAL FILM UNDER GRAVITY ON A VERTICAL WALL

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UDC 536.242: 532.62

Semiempirical turbulence theory has been used in the study of heat transfer for a liquid-metal film falling under gravity on a smooth vertical wall.

It has been shown [1] that the local heat-transfer coefficient in turbulent flow of a film of liquid under gravity on a flat wall can be defined from

$$\mathrm{Nu}_m = \frac{\mathrm{Re}\sqrt[3]{\eta_\delta}}{4\int\limits_0^{\eta_\delta} \varphi\left(\int\limits_0^{\eta} \frac{q/q_w}{1+\frac{\Pr}{\Pr_t}\frac{\varepsilon_\tau}{\nu}}d\eta\right)d\eta}. \quad (1)$$

If the physical parameters of the film are constant, $q_w = \mathrm{const}$, $\delta_T = \delta_h = \delta$, and if there is no heat transfer to the surrounding medium, then it is found [1] that

$$\frac{q}{q_w} = 1 - \frac{4\int\limits_0^{\eta}\varphi d\eta}{\mathrm{Re}}. \quad (2)$$

If the density of the surrounding medium is neglected, then it is found [2] that for a film on a vertical wall

$$\varphi = 2\int\limits_0^{\eta}\frac{(1-\eta/\eta_\delta)d\eta}{1+\sqrt{1+4\varkappa^2\eta^2N^2(1-\eta/\eta_\delta)}}, \quad (3)$$

where N = 1 for $\eta \geq 25$, while for $\eta \leq 25$ we have

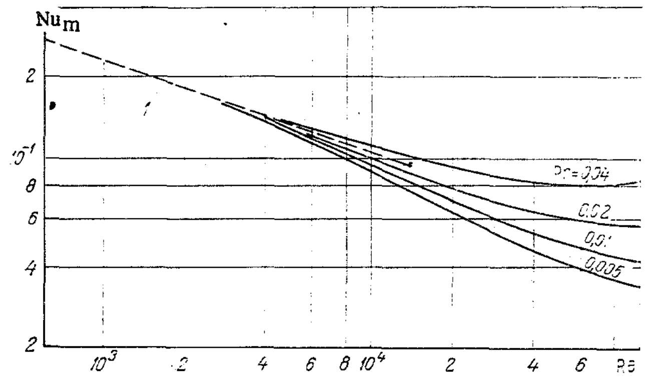

Fig. 1. The dependence of Nu_m on Re and Pr: 1) $\mathrm{Nu}_m = f(\mathrm{Re})$ for laminar flow.

Antanas Sneckus Polytechnic Institute, Kaunas. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 32, No. 2, pp. 197-203, February, 1977. Original article submitted February 20, 1976.

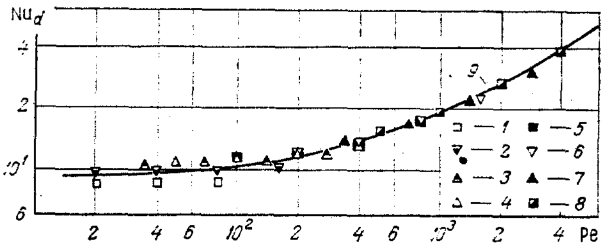

Fig. 2. The dependence of Nu_d on Pe: 1-8) theoretical calculations for Re = 2000, 4000, 7000, 10,000, 20,000, 40,000, 70,000, and 100,000; 9) calculation from (13).

$$N = \exp\left[-\frac{(1-\eta/25)^n}{\sigma}\right]. \tag{4}$$

Here n = 1 and σ = 0.4 for developed turbulent flow (Re > $1.5 \cdot 10^4$) and n = f(Re) and σ = f(Re) for transitional-state flow (Re < $1.5 \cdot 10^4$).

The relationship between η_δ and Re is given by

$$4\int_0^{\eta_\delta} \varphi d\eta = \mathrm{Re}. \tag{5}$$

The turbulent viscosity is defined by

$$\frac{\varepsilon_\tau}{\nu} = \frac{1-\eta/\eta_\delta}{d\varphi/d\eta} - 1. \tag{6}$$

It has been shown [1] that calculations of the heat transfer from the above equations agree well with measurements for water films, i. e., for Pr > 1 if we assume that Pr_t = 0.9 = const in accordance with [3].

Numerous studies have shown that the semiempirical theories of turbulence can also be used for heat transfer for liquid metals flowing in pipes [4-8]. We can therefore assume that (1)-(6) can be applied also to liquid-metal films, provided that Pr_t may be determined.

In general, Pr_t is dependent on Pr and the turbulence, so it is commonly represented in the form $Pr_t(Pr, \varepsilon_\tau/\nu)$, $Pr_t(Pr, \eta)$, or $Pr_t(Pr, \eta, Re)$; it has been found that for liquid metals (Pr ≪ 1) the turbulence is the major factor, and the value may be greater or less than 1 [3, 9-14]. On the other hand, the simple assumption that Pr_t = 1 produces satisfactory agreement with experiment when a correction is applied for the thermal contact resistance at the wall—liquid boundary [4, 5, 7, 8]. The reason is that Pr_t differs substantially from unity only if the turbulence is slight, in which case the turbulent transport is only a small fraction of the total transport, and therefore the value is without much effect on the result. Consequently, it would seem that one can assume Pr_t = 1 = const in order to calculate the transfer for a flowing liquid-metal film, but high accuracy requires the value to be determined from [9, 10], in which the quantity Pr_t is represented as a function of Re. Those data are represented satisfactorily as follows:

$$Pr_t = 4.4\,\mathrm{Re}^{-0.125}. \tag{7}$$

Since (7) relates to a liquid metal in a tube, one has to examine whether it can be used for a film; however, it has been shown [2] that Re expressed in terms of the equivalent diameter is a fairly general characteristic of the flow in a film or tube. For instance, a major hydrodynamic parameter such as the coefficient of friction has a dependence on Re for turbulent flow in a smooth tube almost the same as that for turbulent flow in a film on a smooth wall. This enables us to assume that (7) is applicable also to a liquid-metal film.

This definition was used in numerical calculations from (1)-(6); Fig. 1 shows the result.

Transition from laminar flow to turbulent flow for Pr < 0.04 causes the heat-transfer coefficient to fall more rapidly as Re increases, as was first observed in theoretical studies on the condensation of metal vapors [6] and which arises because the turbulent flow increases the frictional resistance, and this increases the film

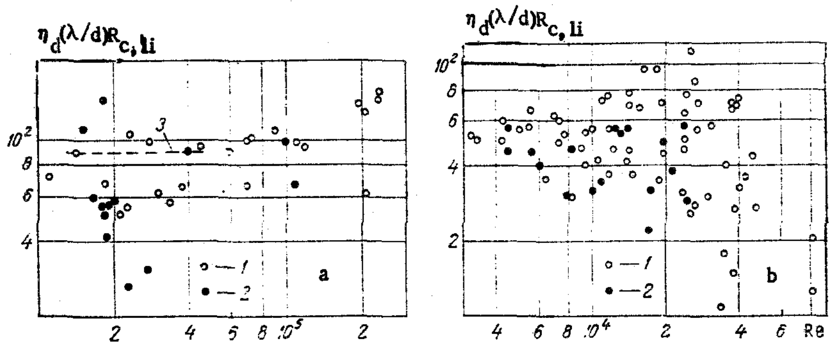

Fig. 3. Thermal contact resistance for a flowing liquid metal: a) in a continuous tube (1: heavy metal [8]; 2: Na—K [8]; 3: Na [15]); b) inner tube composed of ring segments (1: Na—K [8]; 2: Li [8]).

thickness and the thermal resistance of the film. Since the turbulent transport is of minor importance for Pr ≪ 1 when the turbulence is slight, the heat-transfer coefficient becomes less than that for laminar flow. It requires a further increase in Re and the turbulence for the turbulent transport to predominate, in which case α begins to rise again.

The exact values derived in this study differ somewhat from those of [16], since the heat flux varies and the heat-transfer coefficient is calculated not in terms of the total temperature difference across the film, but in terms of the temperature difference between the wall and the mean film temperature.

Theoretical and experimental data are usually processed as Nu = f(Pe) for a liquid metal in a tube; in order to process our data in such a form it is necessary to calculate the Nusselt number in terms of the equivalent film diameter.

If we neglect the surrounding medium entirely, the tangential stress at the vertical wall bearing the film may be expressed as

$$\tau_w = 0.25\rho g d. \tag{8}$$

Then the dimensionless equivalent diameter of the film is

$$\eta_d = \frac{v^* d}{\nu} = \frac{d}{\nu}\sqrt{0.25gd}. \tag{9}$$

It follows from (9) that

$$d = \sqrt[3]{4}\left(\frac{\nu^2}{g}\right)^{\frac{1}{3}}\eta_d^{\frac{2}{3}}. \tag{10}$$

If we substitute for $(\nu^2/g)^{1/3}$ in (10) in terms of Nu_m, we get a relationship between the latter and Nu_d:

$$Nu_d = 1.587\eta_d^{\frac{2}{3}} Nu_m \tag{11}$$

For a film flowing on a flat wall, $d = 4\delta$ and $\eta_d = 4\eta_\delta$, so the relationship between η_d and Re is given by (5); it has been shown [1] that this relationship is fitted by the empirical relation

$$\eta_d = 32 + 0.127\,Re^{0.92}. \tag{12}$$

Then (11), (12), and the data of Fig. 1 serve to define Nu_d = f(Pe), which is shown in Fig. 2 and which can be represented closely by the empirical equation

$$Nu_d = 9 + 0.04\,Re^{0.8}. \tag{13}$$

These results correspond to the case where there is no thermal contact resistance R_c at the wall—liquid boundary; if R_c is finite, the heat transfer may be much lower, as occurs when a liquid-metal coolant is contaminated by gaseous or solid impurities. It is found [8] that suspended particles tend to accumulate near the wall in a liquid metal, and the turbulent mixing leaves a persistent layer of impurities in the viscous sublayer, whose thickness is proportional to the thickness of the latter. This is confirmed by measurements on liquid metals flowing in tubes, which show that R_c decreases as Re increases and as the thickness of the viscous layer falls [8, 15]. The results also show that R_c at first increases with the total impurity content, but subsequently becomes virtually independent of the latter. This is ascribed [8] to saturation of the viscous sublayer with impurities and is confirmed by the observation that the total concentration giving the limiting thermal resistance ($R_{c,\,li}$) is dependent on Re. For instance, it is found [15] that this limit is obtained for a total O_2 concentration of about 0.07% by weight for oxygen in flowing sodium provided that Pe = 1000 (Re ≃ 150,000), whereas the limit is attained only at about 0.1% O_2 if Pe = 100 (Re ≃ 15,000), so the thickness of the viscous sublayer increases as Re falls, and therefore the layer requires more impurity for saturation.

Therefore, since the contact resistance for a liquid metal in a tube is determined mainly by the hydrodynamics of the flow, we can estimate the limiting contact resistance for a turbulent liquid-metal film.

If this $R_{c,\,li}$ is due to a persistent layer of impurities near the surface, whose thickness is proportional to the thickness of the viscous sublayer, then we can assume that

$$R_{c,\,li} = c\frac{\delta_v}{\lambda}. \tag{14}$$

Further, if we assume that η_v = const as a first approximation (the Nikuradze—Kármán three-layer scheme gives $\eta_v = 5$), and if we bear in mind that $\delta_v/d = \eta_v/\eta_d$, then we have from (14) that

$$\frac{R_{c,\,li}\lambda\eta_d}{d} = \text{const}. \tag{15}$$

It is readily shown that the relationship between η_d and Re for turbulent flow in a tube is

$$4\int\limits_0^{\eta_d/2}(1 - 2\eta/\eta_d)\varphi d\eta = \text{Re}. \tag{16}$$

If we use a three-layer Nikuradze—Kármán scheme: $\varphi = \eta$ for $0 \le \eta \le 5$, $\varphi = 5\ln\eta - 3.05$ for $5 \le \eta \le 30$, and $\varphi = 2.5\ln\eta + 5.5$ for $30 \le \eta \le 0.5\eta_d$, then integration of (16) and certain other operations give us

$$\text{Re} = 2.5\eta_d\ln\eta_d - 255. \tag{17}$$

Numerical calculations readily show that (12) and (17) give almost identical results, so (10) indicates that d/η_d has the same Re dependence for turbulent flow in a smooth tube and a turbulent film on a smooth vertical wall.

Velocity-distribution measurements on films [2] show that the dimensionless thickness of the viscous sublayer can be taken as being the same as for a flow in a tube.

Therefore, if we assume that the constant of proportionality c in (14) is the same in both cases, it follows that $R_{c,\,li}\lambda\eta_d/d$ should also be the same.

Figure 3 shows this complex as a function of Re; this has been derived from (17) and measurements on the limiting contact resistance for liquid metals in tubes [8, 15]. The results of Fig. 3a correspond to continuous tubes, while those of Fig. 3b relate to a tube composed of ring segments; it is clear that in both cases $R_{c,\,li}\lambda\eta_d/d$ scarcely varies with Re, while the sharp fall for Re > 30,000 in the ring-segment tube is due [8] to the thickness of the laminar sublayer becoming less than the height of the roughness Re > 30,000, particularly as the elements may not be exactly coaxial.

The heat-transfer coefficient when $R_{c,\,li}$ is attained is

$$\alpha_1 = \frac{1}{\frac{1}{\alpha} + R_{c,\,li}}. \tag{18}$$

Then

$$\frac{1}{\text{Nu}_{d_1}} - \frac{1}{\text{Nu}_d} = \frac{R_{c,\,li}\lambda}{d}. \tag{19}$$

One can assume for the most unfavorable heat-transfer conditions (Fig. 3a) that on average $R_{c,\,li}\lambda\eta_d/d \simeq 90$, and then (19) gives

$$\mathrm{Nu}_{d_1} = \frac{\mathrm{Nu}_d}{1 + \frac{90}{\eta_d}\mathrm{Nu}_d}, \tag{20}$$

where η_d and Nu_d are defined by (12) and (13).

Then (13) and (20) define the upper and lower bounds to the heat-transfer coefficient for a liquid-metal film flowing under gravity on a verticle smooth wall. The results from these equations show that film contamination can reduce the heat transfer by 30-70% at high Re or by a much more substantial factor at small Re.

Unfortunately, I am unaware of any measurements on the flow and heat transfer in liquid-metal films that could be invoked for comparison with these theoretical results.

NOTATION

$Nu_m = \alpha^3\sqrt{\nu^2/g/\lambda}$, modified Nusselt number; $Nu_d = \alpha d/\lambda$, Nusselt number; $Re = \bar{w}d/\nu = 4\Gamma/(\rho\nu)$, Reynolds number; $Pe = Re \cdot Pr$, Peclet number; $Pr = \nu/a$, Prandtl number; $Pr_t = \varepsilon_\tau/\varepsilon_q$, turbulent Prandtl number; $\eta = v^*y/\nu$, dimensionless distance from wall; $\eta_\delta = v^*\delta/\nu$, dimensionless thickness of film; $\eta_v = v^*\delta_v/\nu$, dimensionless thickness of viscous sublayer; $\eta_d = v^*d/\nu$, dimensionless equivalent diameter; $\varphi = w/v^*$, dimensionless velocity; $v^* = \sqrt{\tau_w/\rho}$ dynamic velocity; $d = 4F/U$, equivalent diameter; α, local heat-transfer coefficient for pure film; ν, kinematic viscosity; λ, thermal conductivity; g, acceleration due to gravity; w, $\bar{w}$, local and mean velocities; Γ covering density; ρ, density; a, thermal diffusivity; ε_τ, turbulent viscosity; ε_q, turbulent thermal diffusivity; y, distance from wall; δ, mean film thickness; δ_v, viscous sublayer thickness; δ_T, thermal boundary-layer thickness; δ_h, hydrodynamic boundary-layer thickness; F, cross-sectional area of flow; U, wetted perimeter; q, specific heat flux; τ, tangential stress; $\varkappa = 0.4$, turbulence constant; subscript w denotes parameters at wall.

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